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A Linear Separability Criterion for Sets of Euclidean Space

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Abstract

We prove new theorems which describe a necessary and sufficient condition for linear (strong and non-strong) separability and inseparability of the sets in a finite-dimensional Euclidean space. We propose a universal measure for the thickness of the geometric margin (both the strong separation margin (separator) and the margin of unseparated points (pseudo-separator)) formed between the parallel generalized supporting hyperplanes of the two sets which are separated. The introduced measure allows comparing results of linear separation obtained by different techniques for both linearly separable and inseparable sets. An optimization program whose formulation provides a maximum thickness of the separator for the separable sets is considered. When the sets are inseparable, the same solver is guaranteed to construct a pseudo-separator with a minimum thickness. We estimate the distance between the convex and closed sets. We construct a cone of generalized support vectors for hyperplanes, each one of which linearly separates the considered sets. The interconnection of the problem of different types of linear separation of sets with some related problems is studied. © 2012 Springer Science+Business Media, LLC.

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Keywords

Cone of support vectors, Distance between the sets, Generalized support vector, Generalized supporting hyperplane, Projection, Pseudo-separator, Separator, Thickness of the separator (pseudo-separator)